

**Solution de l'exercice 3.1 page : 62**

$$M = \begin{pmatrix} -6 & 5 & 3 \\ -8 & 7 & 4 \\ -2 & 1 & 1 \end{pmatrix}$$

1-Polynôme caractéristique

$$\text{Det}(M - \lambda I) = \begin{vmatrix} -6-\lambda & 5 & 3 \\ -8 & 7-\lambda & 4 \\ -2 & 1 & 1-\lambda \end{vmatrix} = \begin{vmatrix} -6 & 5 & 3 \\ 0 & 3-\lambda & 4\lambda \\ -2 & 1 & 1-\lambda \end{vmatrix}$$

On développe suivant la 2ème ligne

$$\text{Det}(M - \lambda I) = (3-\lambda) \begin{vmatrix} -6-\lambda & 3 \\ -2 & 1-\lambda \end{vmatrix} - 4\lambda \begin{vmatrix} -6 & 5 \\ -2 & 1 \end{vmatrix} = (3-\lambda)(-6-\lambda+6)+4\lambda(-6+5+10)$$

$$\text{Det}(M - \lambda I) = (3-\lambda)(-6+6\lambda-\lambda+\lambda^2+6) = -4\lambda(-6-\lambda+10)$$

$$\text{Det}(M - \lambda I) = (3-\lambda)(\lambda^2+5\lambda)-4\lambda(4-\lambda) = \lambda(3-\lambda)(\lambda+5)-4\lambda(4-\lambda) = \lambda[3\lambda+15+\lambda^2-5\lambda-16+4\lambda] = \lambda[-\lambda^2+2\lambda-1] = -\lambda[\lambda^2-2\lambda+1] = -\lambda[\lambda-1].$$

Donc le polynôme caractéristique :

$$\lambda(\lambda-1)^2 = 0.$$

Les valeurs propres :

$$\lambda=0(\text{simple}) : \lambda=1(\text{double})$$

$$\lambda=0 \quad Mx=0x=0$$

$$\begin{pmatrix} -6 & 5 & 3 \\ -8 & 7 & 4 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -6x + 5y + 3z = 0 & (1) \\ -8x + 7y + 4z = 0 & (2) \\ -2x + y + z = 0 & (3) \end{cases}$$

$$(3) \Rightarrow z = 2x - y$$

En remplace dans (2)

$$-8x + 7y + 4(2x - y) = 0 \Rightarrow -8x + 7y + 8x - 4y = 0 \Rightarrow 3y = 0 \Rightarrow y = 0$$

$$(1) \Rightarrow -6x + 3(2x) = 0 \Rightarrow 0 = 0 \Rightarrow x \in \mathbb{R}$$

$$E_0 = \{(x, 0, 2x) / x \in \mathbb{R}\}$$

$$\lambda=1 \quad M \cdot x = 1 \cdot x = x$$

$$\begin{pmatrix} -6 & 5 & 3 \\ -8 & 7 & 4 \\ -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$-6x + 5y + 3z = 0 \quad (1)$$

$$-8x + 7y + 4z = 0 \quad (2)$$

$$-2x + y + z = 0 \quad (3)$$

$$(3) \Rightarrow -2x + y = 0 \Rightarrow y = 2x$$

$$(2) \Rightarrow -8x + 7(2x) + 4z = 0 \Rightarrow -8x + 14x + 4z = 0 \Rightarrow 4x = -4z \Rightarrow x = -z$$

$$(1) \Rightarrow -6x + 5(2x) + 3(-x) = 0 \Rightarrow x = 0$$

$$E_1 = \{(x, 2x, -x) / x \in \mathbb{R}\}$$

$$(2) - \dim E_0 = 1$$

$$- \dim E_1 = 1$$

$$- \dim E_1 + \dim E_1 = 2 < 3 \text{ par le 2eme critère } M \text{ n'est pas diagonale}$$